PROBABILITY

→ It has $\boxed{2}$ APPROACHES :
1) SUBJECTIVE
2) OBJECTIVE

DEFINITIONS

1) **EXPERIMENT:-** THE performance which produces "certain Results".

2) **RANDOM EXPERIMENT:-** IF the Result of the experiment Depends on "chance only" then Experiment is called "Random experiment".
EX: A coin is Tossed.

3) **EVENT :-** The outcome OF an Experiment is called Event.
Ex: let A be an event OF getting Head.

TYPES OF EVENT

a) **null event**

→ Event having "no sample points".

→ It is denoted by "ϕ".

b) **SIMPLE Or Elementary event**

→ A subset OF a sample space, consisting of a single element.

c) **Mutually Exclusive And Exhaustive Events**

→ IF $A \cup B = S$, then Exhaustive.

$A \cap B = \phi$, then Exclusive.

d) **EQUALLY LIKELY EVENTS/ Mutually symmetric Events/ EQUI -probable Events**

→ When Two events has "EQUAL" chance of occurrence.

Eg: A fair coin is Tossed.
A child is born in a family.

e) **Independent Events**

→ If happening of one event does not effect the occurrence of other.

4) **SAMPLE SPACE: -**

→ The set of all possible outcomes of an Random experiment.

→ It is denoted by "S".

→ Eg: A dice is thrown

→ S:{1,2,3,4,5,6}.

→ n(S) = 6.

APPROACHES TO PROBABILITY:

1) Classical or mathematical or Priori.

2) Emperical or posteriori or statistical

3) Axiomatic

1) Classical or mathematical or priori definition :-

→ Assignment OF Real numbers 0 to 1 to the events defined in a sample spaces is known as probability.

→ $$P(A) = \frac{n(A)}{n(S)} = \frac{\text{favourable outcomes}}{\text{Possible outcomes}}$$

NOTE:
1) P(A) always lies between 0 and 1. ($0 \le x \le 1$)
2) $P(A) = 0$ [Impossible event].
3) $P(A) = 1$ [SURE EVENT].
4) $P(A) + P(A') = 1$ [$A' \Rightarrow$ complement event].
5) $P(A \cap A') = \phi$

2) Emperical or statistical:

If a random experiment is Repeated large number of times.

$$P(A) = \lim_{n \to \infty} \frac{n(A)}{n(S)}$$

3) Axiomatic:

- ➡️ It is dependent on Set theory.
- ➡️ $P(A) \geq 0$
- ➡️ $P(S) = 1$
- ➡️. Mutually Exclusive event $P(A \cap B) = 0$
- ➡️. $P(A \cup B) = P(A) + P(B)$

IMPORTANT SAMPLE SPACE

1) COINS:- $N(S) = 2^n$

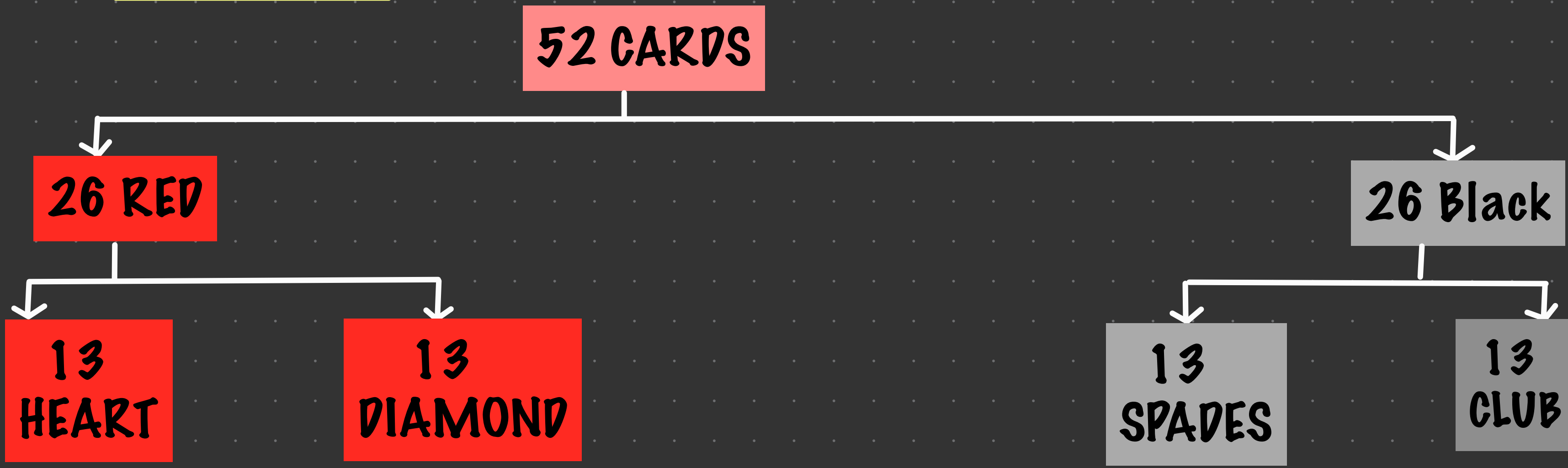
FOR 2 Coins = { HH, HT, TH, TT }
FOR 3 Coins = { HHH,HHT,HTH,HTT,TTH,THT,TTT,THH }

2) DICE:- $N(S) = 6^n$

FOR 1 Die = {1,2, 3, 4, 5, 6 }
FOR 2 Die = {(1,1),(1,2)....................(6,6) }

NOTE : Set Of Doublets = {(1,1),(2,2), (3,3), (4,4), (5,5), (6,6) }

3) CARDS:-



NO OF SUITS = 4
No OF CARDS IN EACH SUIT = 13 (ace,2,3......10,K,Q,J)
No of King, Queen, Jack, Ace cards = 4
Total face cards = 12
Total Picture/ honor cards = 16 (face + Ace)

4) NO OF Childrens in family:-

→ set of 2 Childrens = {BB, BG, GB, GG}

→ set of 3 Children= {BBB, BBG, BGB, BGG, GBG, GBB, GGB, GGG}

5) LEAP YEAR / NON-LEAP YEAR:-

1) LEAP YEAR = 366 days (52 Weeks +2 extra days)

$$P(\text{Any day}) = \frac{2}{7}$$

(2) Non-Leap year = 365 days (52 weeks +1 extra day)

$$P(\text{Anyday}) = \frac{1}{7}$$

PERMUTATION / COMBINATION

PERMUTATION	COMBINATION
→ Arrangement	→ selection
→ $\boxed{{}^{n}P_{r} = \frac{n!}{(n-r)!}}$	→ $\boxed{{}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}}$
→ ${}^{4}P_{2} = 4 \times 3 = 12$	→ ${}^{4}C_{2} = \frac{4 \times 3}{2 \times 1} = 6$

ALGEBRA OF EVENTS

1) $P(A) + P(A') = 1$

2) ADDITION THEOREM :-

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

3) DIFFERENCES :-

$P(A \cap B') = P(A-B) = P(A) - P(A \cap B)$ (A but not B)

$P(B \cap A') = P(B-A) = P(B) - P(A \cap B)$ (B but not A)

4) SYMMETRIC :-

$$P(A \triangle B) = P(A) + P(B) - 2P(A \cap B)$$

5) DEMORGAN'S LAW :-

$$P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$$

$$P(A' \cup B') = P(A \cap B)' = 1 - P(A \cap B)$$

NOTE: atleast (union)
Both/but (intersection)

conditional probability

→ Consider, A Random experiment of Tossing a coin Twice.

$S = \{ HH, HT, TH, TT \}$ $\boxed{n(S) = 4}$

let Event E ≡ Exactly one Head

$E = \{ HT, TH \}$ $\boxed{n(E) = 2}$

let Event f ≡ atleast one Head

$f = \{ HT, TH, HH \}$ $\boxed{n(f) = 3}$

Conditional probability OF Event E : Probability that is based on some conditions.

Condition: probability of Event E given that Event F has already being occurred.

$$\therefore \boxed{P(E/F) = \frac{\text{favourable outcomes}}{\text{possible outcomes}} = \frac{P(E \cap F)}{P(F)}}$$

→ Probability of E given that F has already being occurred.

what is the probability of getting Exactly one head given that atleast one Head occurred?

$$\Rightarrow P(E/F) = \frac{2}{3}$$

Statement: If E and F are two events Associated with the sample space of a Random experiment, the conditional probability of Event E given that F has already occurred. (Denoted by E/F)

$$\therefore \boxed{P\left(E/F\right) = \frac{P(E \cap F)}{P(F)}}$$

Eg: A Family has 2 Children. What is the probability that both the children are boys given that atleast one of them is a boy?

$\Rightarrow \quad S = \{BB, BG, GB, GG\}$

$\boxed{n(S) = 4}$

let E: Both the children are boys

$E = \{BB\}$

$\boxed{n(E) = 1}$

let f: Atleast One of them is a boy

$f = \{Bg, gB, BB\}$

$\boxed{n(f) = 3}$

$\therefore$ By using conditional probability

$$P(E/f) = \frac{P(E \cap f)}{P(f)}$$

$$\boxed{P(E/f) = \frac{1}{3}}$$

INDEPENDENT EVENTS

→ If the occurrence of one event $\boxed{\text{does not effect}}$ the occurrence of other , then the events are called Independent events.

1) $P(A \cap B) = P(A) \cdot P(B)$
2) $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$
3) $P(A' \cap B') = P(A') \cdot P(B')$
4) $P(A' \cap B' \cap C') = P(A') \cdot P(B') \cdot P(C')$

Multiplication theorem :

If A & B are events of S then probability of occurrence of both A&B is

$$P(A \cap B)$$

- $P(A) \cdot P(B/A)$
- $P(B) \cdot P(A/B)$

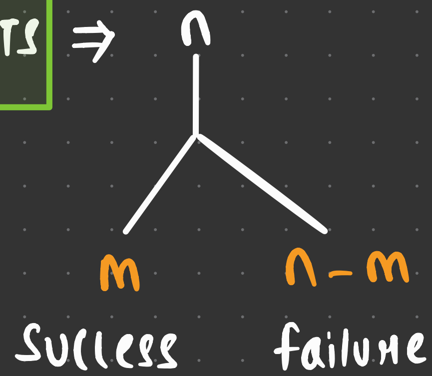
$\underbrace{P(A) \cdot P(B/A) \qquad P(B) \cdot P(A/B)}$

Theory of compound Probability

- $P(A) \cdot P(B)$ → Independent Events
- 0 → $(A \cap B = \phi)$
- Addition Theorem → $P(A)$, $P(B)$, $P(A \cap B)$ given

ODDS IN FAVOUR AND AGAINST AN EVENT

$\boxed{\text{SAMPLE POINTS}} \Rightarrow$



$\therefore \boxed{P(\text{success}) = \dfrac{m}{n}}$ $\quad \boxed{P(\text{failure}) = \dfrac{n-m}{n}}$

1. <u>Odds in favour</u> : $P(\text{success}) : P(\text{failure})$

i.e. $\boxed{m : n - m}$

② Odd's against any event : P(failure) : P(success)

i.e. $\boxed{n-m : m}$

Eg: The odds in favour are 7 : 5 and odd's against an event are 4 : 3.

$\Rightarrow$ Odd's in favour = 7 : 5

$P(A) = \frac{7}{12}$ $\quad P(A') = \frac{5}{12}$

Odd's against an event = 4 : 3

$P(B') = \frac{4}{7}$ $\quad P(B) = \frac{3}{7}$

Random Variable (X) : it represents all outcomes of a random experiment by real numbers .

Discrete

(How many ?)

Probability mass function helps in Distributing probability among different values of x

Continuous

(How much ?)

Probability density function helps in distributing probability among different intervals of x

Probability mass function (Pmf) :

① $P(X = x) \geq 0$

and

② $\Sigma P(X = x) = 1$

Cumulative distributive function (cdf):

→ Also called as step function

→ $\boxed{F(X = x) = P(X \leq x)}$

Probability density function (pdf):

① $f(x = x) \geq 0$

and

② $\int_{-\infty}^{\infty} f(x)\, dx = 1$

Expected value, variance & S.D :

Prepare 5 columns namely : $\boxed{x_i \quad p_i \quad p_i x_i \quad x_i^2 \quad p_i x_i^2}$

$$\boxed{E(x) = \Sigma p_i x_i}$$

$$\boxed{\begin{aligned} V(x) &= \Sigma p_i x_i^2 - [E(x)]^2 \\ &= E(x^2) - [E(x)]^2 \\ &= E(x-\mu)^2 \end{aligned}}$$

$$S.D(x) = \sqrt{V(x)}$$

Note:

1) if E(x) = +ve
Then probability is in our favour.

2) If E(x) = -ve, then probability is against us

3) If E(x) = 0, then it Is fair or unbiased or neutral

Properties of Expected value :

1) $E(k) = k$

2) $E(k.x) = k.\ E(x)$

3) $E(X+Y) = E(X) + E(Y)$

4) $E(X.Y) = E(X).\ E(Y)$

Properties of Variance :

1) $V(K) = 0$

2) $V(K.x) = K^2 . V(x)$

3) $V(ax \pm by) = V(ax) + V(by)$

$= a^2 . V(x) + b^2 . V(y)$